

Searching for Majorana Fermions in 2D Spin-orbit Coupled Fermi Superfluids at Finite Temperature

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Recent experimental breakthrough in realizing spin-orbit (SO) coupling for cold atoms has spurred considerable interest in the physics of 2D SO coupled Fermi superfluids, especially topological Majorana fermions (MFs) which were predicted to exist at zero temperature. However, it is well known that long-range superfluid order is destroyed in 2D by the phase fluctuation at finite temperature and the relevant physics is the Berezinskii-Kosterlitz-Thouless (BKT) transition. In this Letter, we examine finite temperature effects on SO coupled Fermi gases and show that finite temperature is indeed necessary for the observation of MFs. MFs are topologically protected by a quasiparticle energy gap which is found to be much larger than the temperature. The restrictions to the parameter region for the observation of MFs have been obtained.

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A new research direction in low dimensional condensed matter physics that attracts much recent attention is the study of two-dimensional (2D) topological quantum states of matter (e.g., fractional quantum Hall effects, chiral p -wave superfluids/superconductors, *etc.*) that support exotic quasiparticle excitations (named anyons) with Abelian or non-Abelian exchange statistics [1–8]. For instance, it has been shown recently that a topological cold atom superfluid may emerge from an ordinary 2D s -wave Fermi superfluid in the presence of two additional ingredients: spin-orbit (SO) coupling and Zeeman field [9]. Such topological superfluids can host Majorana fermions (MFs), non-Abelian anyons which are their own antiparticles, and may have potential applications in fault-tolerant topological quantum computation [1].

On the experimental side, 2D degenerate s -wave Fermi gases have been realized using highly anisotropic pancake-shaped trapping potential [10–13]. Furthermore, SO coupling and Zeeman field for cold atoms have been generated in recent pioneer experiments through the coupling between cold atoms and lasers [14–19]. It seems therefore that MFs are tantalizingly close to experimental reach in degenerate Fermi gases. Inspired by the experimental achievements, there have been extensive theoretical efforts for understanding the physics of SO coupled Fermi gases, particularly the mean-field crossover from the Bardeen-Cooper-Schrieffer (BCS) superfluids to Bose-Einstein condensation (BEC) of molecules [20–33]. While the mean-field theory works qualitatively well in 3D, it may not yield relevant physics in 2D at finite temperature. For instance, differing from the mean-field prediction, there is no long-range superfluid order in 2D at finite temperature due to phase fluctuations [34]. At finite temperature, the relevant physics is the BKT transition [35, 36] with a characteristic temperature T_{BKT} , below which free vortex-antivortex (V-AV) pairs are formed

spontaneously. When the temperature is further lowered below another critical temperature T_{Vortex} , the system forms a square V-AV lattice [37, 38]. Because MFs only live in the cores of spatially well separated vortices, it is crucial to study the dependence of T_{BKT} and T_{Vortex} on the SO coupling strength in 2D Fermi superfluids.

In this Letter, by taking account of phase fluctuations in the finite temperature quantum field theory, we investigate the finite temperature properties of SO coupled degenerate Fermi gases en route to clarifying some crucial issues for the experimental observation of MFs in such systems. Our main results are the following:

(I) We show that, quite unexpectedly, the SO coupling reduces both T_{Vortex} and T_{BKT} , despite it enhances the superfluid order parameter Δ .

(II) In the pseudogap phase ($T > T_{\text{BKT}}$), the Fermi gas lacks phase coherence. While in the vortex lattice phase ($T < T_{\text{Vortex}}$), the short distance between neighboring vortices may induce a large tunneling between vortices that destroys the Majorana zero energy states [39]. Therefore MFs can be observable only in the cores of naturally present V-AV pairs [40] in the temperature region $T_{\text{Vortex}} < T < T_{\text{BKT}}$, instead of intuitively expected zero temperature.

(III) MF in a vortex core is protected by a quasiparticle energy gap $\gtrsim \Delta^2/2E_F$ (E_F is the Fermi energy), which is found to be much larger than the experimental temperature. Such a large energy gap greatly reduces the occupation probability of non-topological excited states in the vortex core due to finite temperature, and ensures the topological protection of MFs.

(IV) The restrictions to the parameter region for the observation of MFs have been obtained. We also propose a simple strategy for finding experimental parameters for the observation of MFs.

We consider a 2D degenerate Fermi gas in the presence

of a Rashba type of SO coupling and a perpendicular Zeeman field. In experiments, 2D degenerate Fermi gases can be realized using a 1D deep optical lattice with a potential $V_0 \sin^2(2\pi z/\alpha_w)$ along the third dimension, where the tunneling between different layers is suppressed completely [10–13]. The Rashba SO coupling and Zeeman field can be realized using adiabatic motion of atoms in laser fields [14, 15]. The Hamiltonian for this system can be written as ($\hbar = K_B = 1$)

$$H = H_F + H_{\text{soc}} + H_I, \quad (1)$$

where the single atom Hamiltonian $H_F = \sum_{\mathbf{k}, \sigma=\uparrow, \downarrow} (\epsilon_{\mathbf{k}} - \mu_{\sigma}) C_{\mathbf{k}\sigma}^{\dagger} C_{\mathbf{k}\sigma}$, $C_{\mathbf{k}\sigma}^{\dagger}$ is the creation operator for a fermion atom with momentum \mathbf{k} and spin σ , $\epsilon_{\mathbf{k}} = k^2/2m$, m is the atom mass, $\mu_{\uparrow} = \mu + h$, $\mu_{\downarrow} = \mu - h$, μ is the chemical potential, and h is the Zeeman field. The Hamiltonian for the Rashba type of SO coupling is $H_{\text{soc}} = \alpha \sum_{\mathbf{k}} [(k_y - ik_x) C_{\mathbf{k}\uparrow}^{\dagger} C_{\mathbf{k}\downarrow} + (k_y + ik_x) C_{\mathbf{k}\downarrow}^{\dagger} C_{\mathbf{k}\uparrow}]$. The interaction between atoms is described by $H_I = -g \sum_{\mathbf{k}} C_{-\mathbf{k}\uparrow}^{\dagger} C_{\mathbf{k}\downarrow}^{\dagger} C_{\mathbf{k}\downarrow} C_{-\mathbf{k}\uparrow}$, where the effective regularized interaction parameter $1/g = \sum_{\mathbf{k}} 1/(2\epsilon_{\mathbf{k}} + E_b)$ for a 2D Fermi gas [28, 31, 32, 41]. In experiments, the binding energy E_b can be controlled by tuning the s -wave scattering length or the barrier height V_0 along the z direction. Small and large E_b correspond to the BCS and BEC limit, respectively [42].

The finite temperature properties of the 2D Fermi gas are obtained using finite temperature quantum field theory, where the action for the Hamiltonian (1) is $S_V = \int_0^{\beta} d\tau [\sum_{\mathbf{k}, \sigma} C_{\mathbf{k}\sigma}^{\dagger} \partial_{\tau} C_{\mathbf{k}\sigma} + H]$ with $\beta = 1/T$. Introducing the standard Hubbard-Stratonovich transformation with the mean field superfluid order parameter $\phi = g \sum_{\mathbf{k}} \langle C_{\mathbf{k}\downarrow} C_{-\mathbf{k}\uparrow} \rangle$ and integrating out the fermion degrees of freedom, we have the partition function $Z = \int D\phi D\phi^* \exp(-S_{\text{eff}})$ with the effective action $S_{\text{eff}} = \int_0^{\beta} d\tau (g^{-1} |\phi|^2 + \zeta_{\mathbf{k}}) - \frac{1}{2} \text{Tr}[\ln G^{-1}]$. Here

$$G^{-1} = \begin{pmatrix} \partial_{\tau} + \zeta_{-, \mathbf{k}} & -\alpha k_- & 0 & -\phi \\ -\alpha k_+ & \partial_{\tau} + \zeta_{+, \mathbf{k}} & \phi & 0 \\ 0 & \phi^* & \partial_{\tau} - \zeta_{-, \mathbf{k}} & -\alpha k_+ \\ -\phi^* & 0 & -\alpha k_- & \partial_{\tau} - \zeta_{+, \mathbf{k}} \end{pmatrix},$$

is the inverse Nambu matrix under the Nambu basis $\Psi(\mathbf{k}) = (C_{\mathbf{k}\uparrow}, C_{\mathbf{k}\downarrow}, C_{-\mathbf{k}\downarrow}^{\dagger}, C_{-\mathbf{k}\uparrow}^{\dagger})^T$, the symbol Tr denotes the trace over momentum, imaginary time, and the Nambu indices, $k_{\pm} = (k_y \pm ik_x)$ and $\zeta_{\pm, \mathbf{k}} = (\epsilon_{\mathbf{k}} - \mu) \pm h = \zeta_{\mathbf{k}} \pm h$. The superfluid order parameter Δ is obtained from the saddle point of the effective action (i.e., $\frac{\partial S_{\text{eff}}}{\partial \phi^*} |_{\phi=\Delta} = 0$), which yields the gap equation

$$\sum_{\mathbf{k}} \frac{1}{(2\epsilon_{\mathbf{k}} + E_b)} = \frac{1}{2} \sum_{\mathbf{k}, l=\pm} \left[\lambda_l E_{l, \mathbf{k}}^{-1} \tanh(\beta E_{l, \mathbf{k}}/2) \right]. \quad (2)$$

Here the quasiparticle energy spectrum $E_{\pm, \mathbf{k}} = \sqrt{E_{\mathbf{k}}^2 + \alpha^2 k^2 + h^2 \pm 2A}$, $A = \sqrt{\alpha^2 \zeta_{\mathbf{k}}^2 k^2 + h^2 E_{\mathbf{k}}^2}$, $\lambda_{\pm} = \frac{1}{2}(1 \pm h^2/A)$, $E_{\mathbf{k}}^2 = \zeta_{\mathbf{k}}^2 + |\Delta|^2$, and $k^2 = k_x^2 + k_y^2$.

In 2D Fermi gases, it is well known that long-range superfluid order can be destroyed by phase fluctuations of the order parameter at any finite temperature [34]. To study the phase fluctuations in SO coupled Fermi gases, we set $\phi = \Delta e^{i\theta}$ following the standard procedure, where θ is the superfluid phase around the saddle point (determined by Eq. (2)) that varies slowly in position and time spaces. The superfluid phase can be decoupled from the original Green's function through a unitary transformation $UG^{-1}(\theta)U^{\dagger} = G_0^{-1} - \Sigma$, where $U = \exp(iM\theta/2)$, and $M = \text{diag}(1, 1, -1, -1)$. $\Sigma = \tau_3 \left(\frac{i\partial_{\tau}\theta}{2} + \frac{(\nabla\theta)^2}{8m} \right) - I \left(\frac{i\nabla^2\theta}{4m} + \frac{i\nabla\theta \cdot \nabla}{2m} \right) + \frac{\alpha}{2} (\tau_3 \sigma_x \partial_y \theta - I \sigma_y \partial_x \theta)$ is the corresponding self-energy, σ_i and τ_i are Pauli matrices in the Nambu space. G_0^{-1} is the Green's function at $\phi = \Delta$. The effective action can be written as $S_{\text{eff}} = S_0(\Delta) + S_{\text{fluc}}(\nabla\theta, \partial_{\tau}\theta)$ [43], where $S_{\text{fluc}}(\nabla\theta, \partial_{\tau}\theta) = \text{tr} \sum_{n \geq 1} \frac{1}{n} (G_0 \Sigma)^n$. Expanding Σ up to the leading order ($n = 2$), we have

$$S_{\text{fluc}} = \frac{1}{2} \int d^2 \mathbf{r} [J(\nabla\theta)^2 + P(\partial_{\tau}\theta)^2 - Q(i\partial_{\tau}\theta)] \quad (3)$$

where $J = \frac{1}{4m} \sum_{\mathbf{k}} (n_{\mathbf{k}} - \sum_{l=\pm} (\mathcal{J}_{1,l} + \mathcal{J}_{2,l}))$ represents the phase stiffness or superfluid density. $n_{\mathbf{k}} = 1 - \sum_{l=\pm} \frac{\xi(1+l\eta)}{2E_{l, \mathbf{k}}} \text{Tanh}(\frac{\beta E_{l, \mathbf{k}}}{2})$, $\mathcal{J}_{1,l} = \frac{m\alpha^2}{2E_{l, \mathbf{k}}} [(1 - \frac{k^2\alpha^2\xi^2}{2A^2}) + l(\frac{\Delta^2+\xi^2}{2A} + \frac{h^2((\Delta^2+\xi^2)^2+k^2\alpha^2\Delta^2)}{2A^3})] \text{Tanh}(\frac{\beta E_{l, \mathbf{k}}}{2})$, $\mathcal{J}_{2,l} = \frac{k^2\beta}{32m^2} (1 + l \frac{m\alpha^2\xi}{A})^2 \text{Sech}^2(\frac{\beta E_{l, \mathbf{k}}}{2})$, $\eta = (h^2 + k^2\alpha^2)/A$, $P = \sum_{\mathbf{k}, l=\pm} \frac{-\xi^2(\eta+l)^2 + E_{l, \mathbf{k}}^2(1 + lh^2\Delta^2\eta/A)}{8E_{l, \mathbf{k}}^3} \text{Sech}(\frac{\beta E_{l, \mathbf{k}}}{2}) + \frac{\beta\xi^2(\eta+l)^2}{16E_{l, \mathbf{k}}^2} \text{Sech}^2(\frac{\beta E_{l, \mathbf{k}}}{2})$, and $Q = \sum_{\mathbf{k}} n_{\mathbf{k}}$. We have checked that the expressions for J , P and Q reduce to previous results in different limits [41, 44–46].

The phase θ can be decomposed into a static vortex part $\theta_v(\mathbf{r})$ and a time-dependent spin-wave part $\theta_{\text{sw}}(\mathbf{r}, \tau)$. As a consequence, the action of the phase fluctuation becomes $S_{\text{fluc}} = S_v + S_{\text{sw}}$ with $S_v = \frac{1}{2} \int d^2 \mathbf{r} J[\nabla\theta(\mathbf{r})]^2$ and $S_{\text{sw}} = \frac{1}{2} \int d^2 \mathbf{r} \{ J[\nabla\theta_{\text{sw}}(\mathbf{r}, \tau)]^2 + P[\partial_{\tau}\theta_{\text{sw}}(\mathbf{r}, \tau)]^2 - Q[i\partial_{\tau}\theta_{\text{sw}}(\mathbf{r}, \tau)] \} = \sum_{\mathbf{k}} \ln[1 - \exp(-\beta\omega_{\mathbf{k}})]$, where $\omega_{\mathbf{k}} = c|\mathbf{k}|$, $c = \sqrt{J/P}$ is the speed of the spin wave [44]. Note that Eq. (3) is *exactly* the same as the effective action for the 2D Heisenberg XY model [35, 36] with different J , P and Q , therefore the BKT transition temperature

$$T_{\text{BKT}} = \frac{\pi}{2} J(\Delta, \mu, T_{\text{BKT}}). \quad (4)$$

Across the BKT temperature, there is a transition from the pseudogap phase (with finite pairing Δ but without phase coherence or superfluidity) to phase coherent V-AV pairs (with both pairing and superfluidity). When the temperature is further lowered, free V-AV pairs form a tightly bounded V-AV lattice below another critical temperature [37, 38],

$$T_{\text{Vortex}} = 0.3 J(\Delta, \mu, T_{\text{Vortex}}). \quad (5)$$

The atom density equation can be obtained from the total thermodynamic potential $\Omega = TS_{\text{eff}}$, yielding [47],

$$n = -\partial\Omega/\partial\mu = \sum_{\mathbf{k}} n_{\mathbf{k}} - \beta^{-1} \partial S_{\text{sw}}/\partial\mu, \quad (6)$$

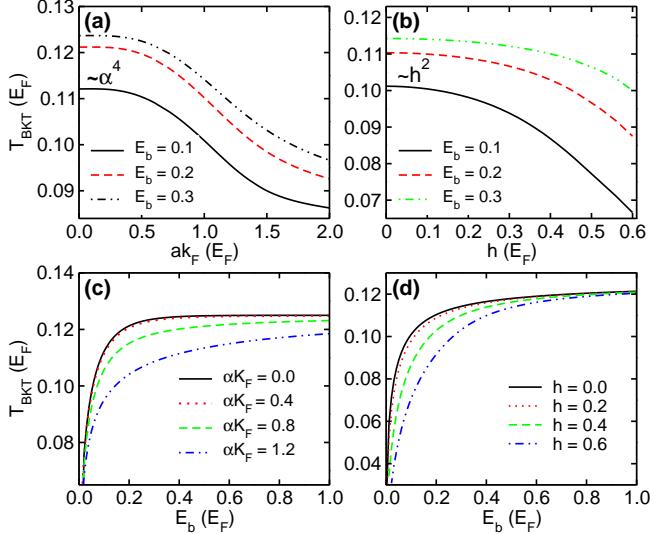


FIG. 1: (Color online) The dependence of the BKT temperature on SO coupling (a), Zeeman field (b), and binding energy (c,d). $h = 0$ in (a) and (c). $\alpha K_F = 1.0 E_F$ in (b) and (d).

where the atom density $n = mE_F/\pi$, and $E_F = \hbar^2 K_F^2/2m$ is the Fermi energy without SO coupling and Zeeman field. The length unit is chosen as the inverse of the Fermi vector K_F^{-1} .

We numerically solve Eqs. (2), (4) and (6) self-consistently, and calculate various physical quantities. In Fig. 1, we plot T_{BKT} with respect to the SO coupling strength αK_F , the Zeeman field h , and the binding energy E_b . We find that, quite surprisingly, T_{BKT} decreases with increasing α , although the superfluid order parameter Δ is enhanced [28]. The unexpected decrease of T_{BKT} has not been found in previous literature [41] and can be understood as follows. For $h = 0$ and $\alpha K_F \ll 1$, the superfluid density

$$J(\alpha) - J(\alpha = 0) \sim - \sum_{\mathbf{k}} \left[\frac{\Delta^2 k^2 \alpha^4}{8E_0^5} + \frac{e^{-\beta E_0} \beta \alpha^2}{8} \right], \quad (7)$$

decreases with increasing α . Here $E_0 = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$. Physically, the increased density of states near the Fermi surface dominates at small α , hence enhances the phase fluctuation and reduces the superfluid density. Note that the next leading term in (7) is $\sim \alpha^6$ for $T = 0$ and $\sim \alpha^4$ for $T \neq 0$, therefore the above analytical result is still valid even for $\alpha K_F \sim E_F$. The Zeeman field is detrimental to the superfluid order parameters, thus further reduces the superfluid density and the BKT temperature, as shown in Fig. 1b and Fig. 1d. Perturbation theory near $h \ll 1$ shows that the change of T_{BKT} is $\sim h^2$, with the coefficient depending strongly on α . Similar features are also found for T_{Vortex} .

In the presence of both SO coupling and Zeeman field, a topological superfluid can emerge from regular s -wave interaction when h is larger than a critical value

$h_c = \sqrt{\mu^2 + \Delta^2}$ [8, 21]. Around h_c , the minimum quasi-particle energy gap occurs at $k = 0$ (i.e., $E_g = E_{-k=0}$), which first closes and then reopens across h_c , allowing the system to change its topological order from a regular s -wave superfluid to a topological superfluid where MFs exist in vortex cores [8]. At finite temperature, the zero temperature topological phase transition becomes a phase crossover. In Fig. 2a, we plot the line $E_g = T$ for a finite h , which is obtained by solving Eqs. (2), (6) and $E_g = T$ self-consistently. The gap closes at a critical E_b^c where $h_c = \sqrt{\mu^2 + \Delta^2} = h$. When $E_b < (>) E_b^c$, $h > (<) h_c$, and the region below the line $E_g = T$ corresponds to the topological (non-topological) superfluid. Above the line $E_g = T$, the temperature is larger than the quasi-particle energy gap and the thermal excitations destroy the topological superfluid. We emphasize that at finite temperature there is no sharp phase transition, but only phase crossover between topological and non-topological superfluids. The line $E_g = T$ is plotted only for guiding purpose and there is no sharp boundary between different phases.

The solid and dash dotted lines in Fig. 2a correspond to T_{BKT} and T_{Vortex} , respectively. We see T_{Vortex} quickly approaches a constant $T_{\text{Vortex}} = 3E_F/40\pi$ with increasing binding energy. In the pseudogap region ($T > T_{\text{BKT}}$), free vortices may exist, but they are not suitable for the observation of MFs due to the lack of phase coherence. While in the vortex lattice region ($T < T_{\text{Vortex}}$), the zero energy modes may break into two normal states with energy splitting $\propto \Delta e^{-R/\xi}$ [39, 48] because of the large tunneling between neighboring vortices in the lattice, where ξ is the coherence length of the superfluid, and R is the intervortex distance. In this region, strong disorder in the tunneling may also lead to Majorana metals [49]. Clearly, the required temperature for observing MFs should be $T_{\text{Vortex}} < T < T_{\text{BKT}}$, instead of the intuitive zero temperature [31, 32]. In Fig. 2a, the shadow region between T_{BKT} and T_{Vortex} gives the possible parameter range for

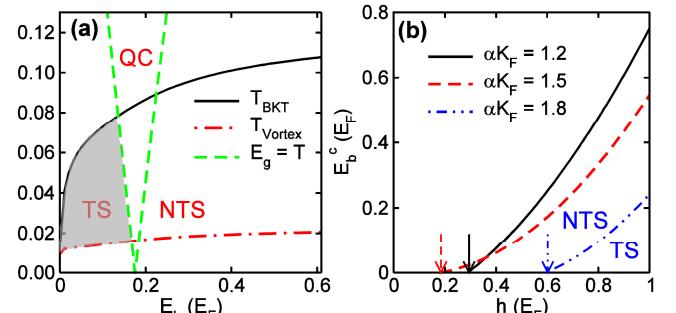


FIG. 2: (Color online). Parameter region for topological superfluids. (a) QC: quantum critical region; TS: topological superfluids; NTS: non-topological superfluids. The shadow region is the possible parameter region for observing MFs. $h = 0.6E_F$, $\alpha K_F = 1.5E_F$. (b) Phase boundary between TS and NTS at $T = 0$. The arrows mark the required minimum Zeeman fields.

the experimental observation of MFs which exist in the cores of the naturally present V-AV pairs in this region. Note that the intervortex distance R is still essential for the observation of MFs in this region. Such distance may be estimated using an analogy between V-AV pairs and the 2D Coulomb gases. It has been shown [54] that the mean-square radius $\langle R^2 \rangle \sim \xi^2 \frac{\pi \beta J-1}{\pi \beta J-2}$, which diverges at $T = T_{\text{BKT}}$ (see Eq. (4)) and the V-AV pair breaks into free vortices. Therefore there should exist a finite temperature regime below T_{BKT} where $R \gg \xi$ and the splitting of the zero energy Majorana states is vanishingly small. Note here that our theory can only capture the average behavior of V-AV pairs and a more delicate theory is still needed to further understand the detailed structure of V-AV pairs.

In Fig. 2b, we plot the parameter region for topological superfluids with respect to the Zeeman field and the binding energy at the zero temperature. Generally, the required critical E_b^c for topological superfluids increases when h increases. Note that here the phase boundary is determined by $h_c = \sqrt{\mu^2 + \Delta^2}$ at $T = 0$, but does not shift much even at finite temperature.

Because MFs can only be observed at finite temperature, there exists a nonzero probability $\sim \exp(-\eta)$ for thermal excitations to non-topological excited states in the vortex core, where the ratio $\eta = \epsilon_m/T$, $\epsilon_m \sim \Delta^2/(2E_F)$ is the minimum energy gap (minigap) [50, 51] in the vortex core that protects zero energy MFs. For the observation of MFs, it is crucially important to have $\eta > 1$, in addition to the requirement of the temperature $T_{\text{Vortex}} < T < T_{\text{BKT}}$ (i.e., $\eta > \eta_{\text{BKT}} \equiv \epsilon_m/T_{\text{BKT}}$ and $\eta < \eta_{\text{Vortex}} \equiv \epsilon_m/T_{\text{Vortex}}$). When $E_b \gg E_F$, we have $\eta_{\text{BKT}} = 8E_b/E_F$ and $\eta_{\text{Vortex}} = 40\pi E_b/E_F$, which means $\eta \gg 1$ for a large E_b . Remarkably, η is also dramatically enhanced by the SO coupling in the BCS side. In Fig. 3, we plot η_{BKT} and η_{Vortex} with respect to E_b for parameters $h = 0.8E_F$ and $\alpha K_F = 1.6E_F$. The vertical line at $E_b \simeq 0.33E_F$ is the boundary between topological and non-topological superfluids. There is a broad region in the BCS side with $\eta_{\text{BKT}} > 1$ even at the highest temperature T_{BKT} . The region can be much larger when the temperature is further lowered to $T = T_{\text{Vortex}}$. The solid circle represents the possible temperature $T = 0.05E_F$ that may be accessible in experiments in the near future [12, 52, 53], yielding $\eta \sim 5.6$ and the thermal excitation probability less than 0.4%. Such a small probability clearly demonstrates that MFs can actually be observed with realistic temperature in experiments. Note that in the presence of SO coupling ϵ_m may be larger than $\Delta^2/(2E_F)$ used for our estimate [51], therefore η could be even larger, which further suppresses thermal excitations.

We illustrate how to find suitable parameters, in particular E_b , for the observation of MFs. Because topological superfluids only exist in the region $h > \sqrt{\mu^2 + \Delta^2}$, we need $|\Delta|^2 \gg |\mu|^2$ to obtain a small h and a large

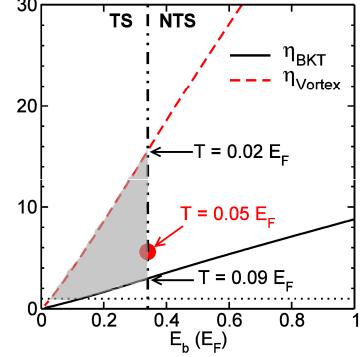


FIG. 3: (Color online) Plot of $\eta = \epsilon_m/T$ as a function of E_b . $h = 0.8E_F$ and $\alpha K_F = 1.6E_F$. Dashed-Dotted line: $E_b^c = 0.33E_F$ as the corresponding boundary between topological superfluids (TS) and non-topological superfluids (NTS) (see the $E_g = T$ line in Fig. 2a for the determination of the phase boundary). Dotted line: $\eta = 1$. Filled circle corresponds to η at $T = 0.05E_F$. The shadow region is the possible parameter region for observing MFs.

Δ . Clearly the BEC limit with large E_b does not work because $|\mu| \sim |E_F - E_b/2| \gg \Delta \sim \sqrt{2E_b E_F}$. While in the BCS side, $|\Delta|^2 \gg |\mu|^2$ can be achieved by choosing suitable SO coupling, Zeeman field and binding energy [28]. To obtain a large quasiparticle minigap in the vortex core (thus a large η) for MFs, the binding energy should be set to be close to the critical E_b^c (see Fig. 3). At the same time, the bulk quasiparticle gap should also be chosen to be much larger than the temperature. Note that in experiments the bulk quasiparticle gap for the topological superfluid can be detected using the recently experimentally demonstrated momentum-resolved photoemission spectroscopy [55].

Finally we briefly compare the cold atomic gases [9] with the semiconductor-superconductor nanostructure where certain signature of MFs has been observed in experiments [56–59]. Both systems share the same ingredients for MFs: SO coupling, Zeeman fields and s-wave pairing, besides the s-wave pairing is through intrinsic s-wave interaction in cold atoms, while externally induced in the nanostructure. Because of high controllability and free of disorder (lacked in the corresponding solid state systems), the topological cold atomic superfluids provide an ideal and promising platform for observing MFs and the associated non-Abelian statistics, which are both fundamentally and technologically important.

In summary, we study the BKT transition in 2D SO coupled Fermi superfluids and find the unexpected decrease of the BKT transition and vortex lattice melting temperatures with increasing SO coupling. We characterize the finite temperature phase diagram for the experimental observation of MFs in this system. Our work not only provides the basis for future study of rich and exotic 2D SO coupled Fermi superfluid physics, but also yields realistic parameter regions for experimentally real-

izing nontrivial topological superfluid states from stable cold atom *s*-wave superfluids.

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